## Circular Motion

1. The Earth takes 1 day to rotate once about its axis. What is the angular velocity of a point on the surface of the Earth?

A $\quad 2.0 \times 10^{-7} \mathrm{rad} \mathrm{s}^{-1}$
B $\quad 7.3 \times 10^{-5} \mathrm{rad} \mathrm{s}^{-1}$
C $\quad 4.4 \times 10^{-3} \mathrm{rad} \mathrm{s}^{-1}$
D $\quad 2.6 \times 10^{-1} \mathrm{rad} \mathrm{s}^{-1}$

Your answer
2. An object of mass $m$ is attached to a string and then whirled in a horizontal circle. The speed of the object is slowly increased from zero. The string breaks when the object has a maximum speed of $1.00 \mathrm{~m} \mathrm{~s}^{-1}$.
The experiment is repeated with an identical string but with an object of mass 1.5 m . The radius of the circle is kept constant.

What is the maximum speed of this object when the string breaks?

A $\quad 0.67 \mathrm{~m} \mathrm{~s}^{-1}$
B $\quad 0.82 \mathrm{~m} \mathrm{~s}^{-1}$
C $\quad 1.22 \mathrm{~m} \mathrm{~s}^{-1}$
D $\quad 1.50 \mathrm{~m} \mathrm{~s}^{-1}$

Your answer
3. For a mass $m$ moving at constant speed $v$ in a circle of radius $r$, the expression for the centripetal force $F$ is

$$
F=\frac{m v^{2}}{r}
$$

Explain the term centripetal force.

4 (a). This question is about the motion of a ball suspended by an elastic string above a bench. The mass of the string is negligible compared to that of the ball. Ignore air resistance.


Fig. 6.1


Fig. 6.2 (not to scale)

In Fig. 6.1 the ball of weight 1.2 N hangs vertically at rest from a point $\mathbf{P}$. The extension of the string is 0.050 m . The string obeys Hooke's law.

In Fig. 6.2 the ball is moving in a horizontal circle of radius 0.045 m around a vertical axis through $\mathbf{P}$ with a period of 0.67 s . The string is at an angle $\theta$ to the vertical. The tension in the string is $T$.

On Fig. 6.2 draw and label one other force acting on the ball.
(b).
i. Resolve the tension $T$ horizontally and vertically and show that the angle $\theta$ is $22^{\circ}$.
ii. Calculate the extension $x$ of the string shown in Fig. 6.2.
$x=$
m [3]
(c). Whilst rotating in the horizontal plane the ball suddenly becomes detached from the string. The bottom of the ball is 0.18 m above the bench at this instant. The ball falls as a projectile towards the bench beneath. Fig. 6.3 shows the view from above.


Fig. 6.3

Calculate the horizontal distance $R$ from the point on the bench vertically below the point $\mathbf{P}$ to the point where the ball lands on the bench.

$$
R=
$$

(d). Returning to the situation shown in Fig. 6.2, state and explain what happens when the rate of rotation of the ball is increased.

5 (a). A particle-accelerator uses a ring of electromagnets to keep protons moving continuously in a circle.
The speed $v$ of the protons depends on the frequency $f$ of rotation of the protons in the circular orbit.
Fig. 22 shows data points plotted on a $v$ against $f$ grid.


Fig. 22
i. Show that the gradient of the graph of $v$ against $f$ is equal to $2 \pi r$, where $r$ is the radius of the circular path of the protons.
ii. Show that $r$ is about 10 m by determining the gradient of the line of best fit through the data points in Fig. 22.
iii. The maximum speed of the protons from this accelerator is $2.0 \times 10^{7} \mathrm{~m} \mathrm{~s}^{-1}$. Calculate the maximum centripetal force $F$ acting on a proton at this speed.

$$
\text { mass of proton }=1.7 \times 10^{-27} \mathrm{~kg}
$$

$$
F=
$$

N [3]
(b). A new particle-accelerator is now built for moving the protons in a circle of a radius 20 m .

The ring of electromagnets for this new accelerator provides the same maximum centripetal force as the accelerator in (a).

Calculate the maximum speed of the protons in this new accelerator.
6. Fig. 22.1 shows the circular track of a positron moving in a uniform magnetic field.


Fig. 22.1
The magnetic field is perpendicular to the plane of Fig. 22.1.
The speed of the positron is $5.0 \times 10^{7} \mathrm{~m} \mathrm{~s}^{-1}$ and the radius of the track is 0.018 m .
State the direction of the force acting on the positron when at point $\mathbf{A}$ and explain why this force does not change the speed of the positron.
7. An astronomer uses a spectrometer and diffraction grating to view a hydrogen emission spectrum from a star. The light is incident normally on the grating.


Fig. 6.1

First order diffraction maxima are observed at angles of $12.5^{\circ}, 14.0^{\circ}$ and $19.0^{\circ}$ to the direction of the incident light as shown in Fig. 6.1.
Two of the wavelengths are $4.33 \times 10^{-7} \mathrm{~m}$ and $4.84 \times 10^{-7} \mathrm{~m}$.
Calculate the wavelength of the third line.
wavelength $=$
m [2]
8. Fig. 21 shows the drum of a washing machine.


Fig. 21
The clothes inside the drum are spun in a vertical circular motion in a clockwise direction.
The washing machine is switched off and the speed of the drum slowly decreases. The clothes at the top of the drum at point $\mathbf{B}$ start to drop off at a certain speed $v$.

At this speed $v$, the normal contact force on the clothes is zero.
Calculate the speed $v$.

```
V=,-------------------------------------ms}\mp@subsup{}{}{-1}[3
```

9. A small object of mass $m$ is placed on a rotating horizontal metal disc at a distance $r$ from the centre of the disc.


The frequency of rotation is adjusted using a motor attached to the disc.
The frequency of rotation of the disc is slowly increased from zero, until the object slips off. At this point, the friction $F$ acting on the object is equal to the centripetal force.

The friction $F$ is given by the expression $F=k m g$, where $k$ is a constant and $g$ is the acceleration of free fall. The constant $k$ has no units.

Show that the frequency $f$ at which the object slips off is given by the equation $f^{2}=\left(\frac{g k}{4 \pi^{2}}\right) \times \frac{1}{r}$.
10. An astronomer uses a spectrometer and diffraction grating to view a hydrogen emission spectrum from a star. The light is incident normally on the grating.


Fig. 6.1

In order to increase the accuracy of the values for wavelength, the student decides to look for higher order diffraction maxima.
i. State how this increases the accuracy.
$\qquad$
$\qquad$
ii. Calculate how many orders $n$ can be observed for the shorter wavelength given in (a).

$$
n=
$$

11. Fig. 22.1 shows the circular track of a positron moving in a uniform magnetic field.


Fig. 22.1
The magnetic field is perpendicular to the plane of Fig. 22.1.
The speed of the positron is $5.0 \times 10^{7} \mathrm{~m} \mathrm{~s}^{-1}$ and the radius of the track is 0.018 m .
Calculate the magnitude of the magnetic flux density of the magnetic field.
magnetic flux density $=$
T [3]
12. Fig. 21 shows the drum of a washing machine.


Fig. 21
The clothes inside the drum are spun in a vertical circular motion in a clockwise direction.
The drum has diameter 0.50 m . The manufacturer of the washing machine claims that the drum spins at $1600 \pm$ 100 revolutions per minute.

Calculate the speed of rotation of the drum and the absolute uncertainty in this value.
13. The London Eye, shown rotating anticlockwise in Fig. 6.1, is a giant wheel which rotates slowly at a constant speed.


Fig. 6.1
Fig. 6.2

Tourists stand in pods around the circumference of the wheel.
The floor remains horizontal at all times.
At time $t=0$, a tourist who has a weight $W$ of 650 N enters a pod at the bottom of the wheel.
Fig. 6.2 shows the forces acting on the tourist at a later time, when the angle between the pod's position and the centre of the wheel is $40^{\circ}$ above the horizontal. $R$ is the normal contact force and $F$ is friction.

Calculate the distance $d$ of the centre of mass of the tourist from the centre of rotation of the London Eye.
The London Eye takes 30 minutes for one rotation.

$$
d=
$$

$\qquad$ .m[3]
14. A rope is attached to a bucket. A man swings the bucket in a horizontal circle of radius 1.5 m . The bucket has a constant speed of $4.8 \mathrm{~m} \mathrm{~s}^{-1}$. The mass of the bucket is 5.0 kg .

i. Calculate the tension $F$ in the rope.
$\qquad$
ii. Calculate the angular velocity $\omega$ of the rotating bucket.
$\omega=$ $\qquad$ rad s ${ }^{-1}$
15. Fluorodeoxyglucose (FDG) is a radioactive tracer often used for PET scans. It contains radioactive fluorine18 , which is a positron-emitter. Some information about FDG and fluorine-18 is given below.

- $9.9 \%$ of the mass of FDG is fluorine-18.
- The half-life of fluorine-18 is 6600 s .
- The molar mass of fluorine-18 is $0.018 \mathrm{~kg} \mathrm{~mol}^{-1}$.

A patient is injected with FDG. The initial activity of FDG is 400 MBq .
Use the information given to calculate the initial mass of FDG given to the patient.
mass $=$ $\qquad$ kg [4]
16. One end of a spring is fixed to a support.

A toy car, which is on a smooth horizontal track, is pushed against the free end of the spring.
The spring compresses. The car is then released. The car accelerates to the right until the spring returns back to its original length.


The car moves with simple harmonic motion as the spring returns to its original length.
 constant of the spring and x is the compression of the spring.

Use the data below to calculate the time $t$ it takes for the spring to return to its original lengthafter the car is released.

- mass of car m=80 g
- force constant k of the spring $=60 \mathrm{~N} \mathrm{~m}^{-1}$.

$$
t=.
$$

17. An astronomer uses a spectrometer and diffraction grating to view a hydrogen emission spectrum from a star. The light is incident normally on the grating.


Fig. 6.1

These three emission lines all arise from transitions to the same final energy level. The part of the energy level diagram of hydrogen relevant to these transitions is shown in Fig. 6.2.
$\qquad$

## Fig. 6.2

i. Draw lines between the energy levels to indicate the transitions which cause the three emission lines and label them with their wavelengths.
ii. There are other possible transitions between the energy levels shown in Fig. 6.2. The least energetic of these produces photons of $4.8 \times 10^{-20} \mathrm{~J}$.

Calculate the wavelength of these photons.
State in which region of the electromagnetic spectrum this wavelength is found.
18. A small object of mass $m$ is placed on a rotating horizontal metal disc at a distance $r$ from the centre of the disc.


The frequency of rotation is adjusted using a motor attached to the disc.
The frequency of rotation of the disc is slowly increased from zero, until the object slips off. At this point, the friction $F$ acting on the object is equal to the centripetal force.

The friction $F$ is given by the expression $F=k m g$, where $k$ is a constant and $g$ is the acceleration of free fall. The constant $k$ has no units.

A student plots a graph of $\lg (f / \mathrm{Hz})$ against $\lg (r / m)$.


For this graph: $y$-intercept $=\frac{1}{2} \times \lg \left(\frac{g k}{4 \pi^{2}}\right)$
Use the graph to determine the constant $k$. Write your answer to 2 significant figures.

19 (a). The International Space Station (ISS) orbits the Earth at a height of $4.1 \times 10^{5} \mathrm{~m}$ above the Earth's surface.

The radius of the Earth is $6.37 \times 10^{6} \mathrm{~m}$. The gravitational field strength $g_{0}$ at the Earth's surface is $9.81 \mathrm{~N} \mathrm{~kg}^{-1}$.
Both the ISS and the astronauts inside it are in free fall.
Explain why this makes the astronauts feel weightless.
(b).
i. Calculate the value of the gravitational field strength $g$ at the height of the ISS above the Earth.

$$
g=\ldots . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . ~ \mathrm{~N} \mathrm{~kg}^{-1}[3]
$$

ii. The speed of the ISS in its orbit is $7.7 \mathrm{~km} \mathrm{~s}^{-1}$. Show that the period of the ISS in its orbit is about 90 minutes.
(c). Use the information in (b)(ii) and the data below to show that the root mean square (r.m.s.) speed of the air molecules inside the ISS is approximately 15 times smaller than the orbital speed of the ISS.

- molar mass of air $=2.9 \times 10^{-2} \mathrm{~kg} \mathrm{~mol}^{-1}$
temperature of air inside the ISS $=20^{\circ} \mathrm{C}$
(d). The ISS has arrays of solar cells on its wings. These solar cells charge batteries which power the ISS. The wings always face the Sun.

Use the data below and your answer to (b)(ii) to calculate the average power delivered to the batteries.

- The total area of the cells facing the solar radiation is $2500 \mathrm{~m}^{2}$.
- $7 \%$ of the energy of the sunlight incident on the cells is stored in the batteries.
. The intensity of solar radiation at the orbit of the ISS is $1.4 \mathrm{~kW} \mathrm{~m}{ }^{-2}$ outside of the Earth's shadow and
- zero inside it.
- The ISS passes through the Earth's shadow for 35 minutes during each orbit.
average power =
W [4]

20. The International Space Station (ISS) circles the Earth at a height of $4.0 \times 10^{5} \mathrm{~m}$.

Its mass is $4.2 \times 10^{5} \mathrm{~kg}$.
The radius of the Earth is $6.4 \times 10^{6} \mathrm{~m}$.
i. Show that the speed of the ISS in orbit is about $8 \mathrm{~km} \mathrm{~s}^{-1}$.
ii. Calculate the total energy of the ISS.
21. The London Eye, shown rotating anticlockwise in Fig. 6.1, is a giant wheel which rotates slowly at a constant speed.


Fig. 6.1
Fig. 6.2

Tourists stand in pods around the circumference of the wheel.
The floor remains horizontal at all times.
At time $t=0$, a tourist who has a weight $W$ of 650 N enters a pod at the bottom of the wheel.
Fig. 6.2 shows the forces acting on the tourist at a later time, when the angle between the pod's position and the centre of the wheel is $40^{\circ}$ above the horizontal. $R$ is the normal contact force and $F$ is friction.

The resultant upward force ( $R-W$ ) on the tourist changes during the 30 minutes of the rotation of the London Eye as shown in Fig. 6.3.


Fig. 6.3
i. Explain why the horizontal force $F$ between the floor and the tourist is necessary.
$\qquad$
$\qquad$
$\qquad$
ii. Draw on Fig. 6.3 the variation of the horizontal force $F$ during the 30 minutes of the anticlockwise rotation of the London Eye. Take forces to the right to be positive.
[2]
iii. Calculate the magnitude of force $F$ when the pod is at the position shown in Fig. 6.2, at $40^{\circ}$ above the horizontal.
22. At an airport, the conveyor belt for suitcases moves at a constant speed of $1.5 \mathrm{~m} \mathrm{~s}^{-1}$. In Fig. 4.1, a suitcase of mass 8.0 kg has reached the line labelled XX'.


Fig. 4.1
Fig. 4.2 shows the situation in vertical cross-section. The frictional force $F$ prevents the suitcase of weight $W$ from sliding to the bottom of the belt.

The normal contact force on the suitcase is $R$.
The belt is inclined at an angle of $30^{\circ}$ to the horizontal.


Fig. 4.2 (not to scale)

Fig. 4.3 shows the suitcase and the forces acting on it at the line labelled $Y^{\prime} \mathbf{Y}^{\prime}$.


Fig. 4.3
The centre of mass of the suitcase is now moving at $1.5 \mathrm{~m} \mathrm{~s}^{-1}$ along a semi-circular arc of radius 2.0 m .
i. Calculate the magnitude of the centripetal force acting on the suitcase.
centripetal force $=$
ii. When the suitcase is at line $Y Y^{\prime}$, the magnitude of force $F$ is larger and the magnitude of force $R$ is smaller than at XX'.
Explain why this is so.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
23. A satellite is in a circular geostationary orbit around the centre of the Earth. The satellite has both kinetic energy and gravitational potential energy.

The mass of the satellite is 2500 kg and the radius of its circular orbit is $4.22 \times 10^{7} \mathrm{~m}$. The mass of the Earth is $5.97 \times 10^{24} \mathrm{~kg}$.

- Describe some of the features of a geostationary orbit.
- Calculate the total energy of the satellite in its geostationary orbit.
$\qquad$
$\qquad$

$\qquad$
$\qquad$
$\qquad$


$\qquad$
$\qquad$
$\qquad$
$\qquad$

$\qquad$
$\qquad$
$\qquad$

$\qquad$

$\qquad$
$\qquad$
$\qquad$
$\qquad$

$\qquad$

24. A satellite moves in a circular orbit of radius 15300 km from the centre of the Earth.
i. State one of the main benefits satellites have on our lives.
$\qquad$
$\qquad$
ii. Calculate the gravitational field strength g at the radius of 15300 km .

$$
g=\ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots . \mathrm{N} \mathrm{~kg}^{-1}
$$

[2]
iii. Calculate the period of the orbiting satellite.
$\qquad$ s
25. * A supply rocket, with its engines shut down, is trying to dock with the International Space Station. Initially it is moving in the same circular orbit above the Earth and at the same speed as the ISS. The two craft are separated by a distance of a few kilometres. The rocket is behind the ISS. It can move closer to the ISS using the following procedure.

The rocket engines are fired in reverse for a few seconds to slow the rocket down. This action causes the rocket to fall into an orbit nearer to the Earth.

After an appropriate time, the rocket engines are fired forwards for a few seconds to move the rocket back into the original orbit closer to the ISS.

Use your knowledge of gravitational forces and uniform motion in a circular orbit to explain the physics of this procedure.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$

$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
26.

Phobos is one of the two moons orbiting Mars. Fig. 17.1 shows Phobos and Mars.


Fig. 17.1
The orbit of Phobos may be assumed to be a circle. The centre of Phobos is at a distance 9380 km from the centre of Mars and it has an orbital speed $2.14 \times 10^{3} \mathrm{~m} \mathrm{~s}^{-1}$.
i.

On Fig. 17.1, draw an arrow to show the direction of the force which keeps Phobos in its orbit.
ii. Calculate the orbital period $T$ of Phobos.

$$
T=
$$

iii. Calculate the mass $M$ of Mars.
27. * A student wishes to test the equation $F=\frac{m v^{2}}{r}$ for a constant force $F$ using a whirling bung in the laboratory.

Describe with the aid of a labelled diagram how an experiment can be conducted, and how the data can be analysed to test the validity of this equation for a constant force.
$\qquad$
$\qquad$
$\qquad$

$\qquad$

$\qquad$

$\qquad$
$\qquad$
$\qquad$
$\qquad$

$\qquad$
$\qquad$
$\qquad$
$\qquad$

$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
28. A binary star is a pair of stars which move in circular orbits around their common centre of mass.

In this question consider the stars to be point masses situated at their centres.
Fig. 3.1 shows a binary star where the mass of each star is $m$. The stars move in the same circular orbit.


Fig. 3.1
i. Explain why the stars of equal mass must always be diametrically opposite as they travel in the circular orbit.
ii. The centres of the two stars are separated by a distance of $2 R$ equal to $3.6 \times 10^{10} \mathrm{~m}$, where $R$ is the radius of the orbit. The stars have an orbital period $T$ of 20.5 days. The mass of each star is given by the equation

$$
m=\frac{16 \pi^{2} R^{3}}{G T^{2}}
$$

where $G$ is the gravitational constant.
Calculate the mass $m$ of each star in terms of the mass $M_{\odot}$ of the Sun.

$$
\begin{aligned}
& 1 \text { day }=86400 \mathrm{~s} \\
& M_{\odot}=2.0 \times 10^{30} \mathrm{~kg}
\end{aligned}
$$

iii. The stars are viewed from Earth in the plane of rotation.

The stars are observed using light that has wavelength of 656 nm in the laboratory. The observed light from the stars is Doppler shifted.

Calculate the maximum change in the observed wavelength $\Delta \lambda$ of this light from the orbiting stars. Give your answer in nm.

